Girraween High School

2016 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using a black or blue pen
- Board - approved calculators may be used
- A laminated reference sheet is provided
- Answer multiple choice questions on the front page
- In questions 11 – 16 start all questions on a separate page and show all relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Pages 5 - 8

10 marks

- Attempt 1 – 10
- Allow about 15 minutes for this section

Section II Pages 9 - 18

90 marks

- Attempt 11 – 16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

1 Find is the value of $log_e 2016$ to three significant figures.

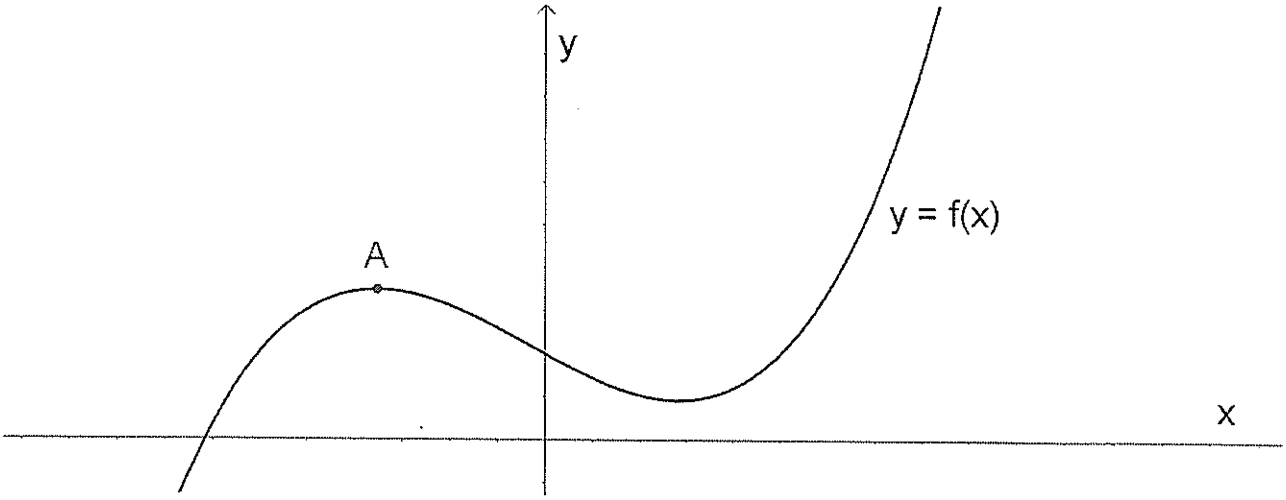
(A) 7.61

(B) 7.60

(C) 7.608

(D) 7.609

2 The graph below shows the maximum stationary point A on the curve $y = f(x)$.



Which of the following is true at point A?

(A) $f'(x) > 0 \ and \ f''(x) = 0$

(B) $f'(x) < 0 \ and \ f''(x) = 0$

(C) $f'(x) = 0 \ and \ f''(x) > 0$

(D) $f'(x) = 0 \ and \ f''(x) < 0$

3 The equation $2x^2 - 5x - 1 = 0$ has roots α and β.
What is the value of $\frac{1}{\alpha} + \frac{1}{\beta}$?

(A) $-\frac{1}{5}$

(B) 5

(C) -5

(D) $\frac{1}{5}$

4 The coordinates of the focus of the parabola $x^2 = 8(y-3)$ are:

(A) $(0,5)$

(B) $(0,1)$

(C) $(5,0)$

(D) $(1,0)$

5 If $x = a(b - \frac{1}{y})$ then

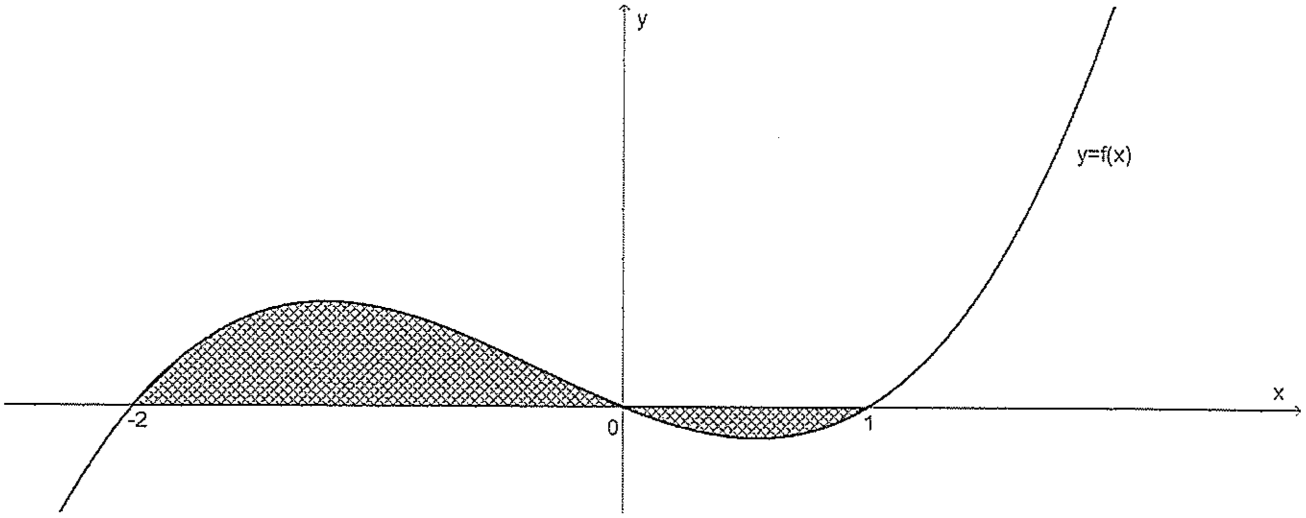
(A) $y = \frac{a}{b-x}$

(B) $y = \frac{a}{ab-x}$

(C) $y = \frac{1}{ab-x}$

(D) $y = \frac{x}{a} - b$

6 Which would give the value of the shaded area?



(A) $\int_0^{-2} f(x)dx + \left|\int_0^1 f(x)dx\right|$

(B) $\int_{-2}^{0} f(x)dx + \left|\int_0^1 f(x)dx\right|$

(C) $\left|\int_0^{-2} f(x)dx\right| + \int_0^1 f(x)dx$

(D) $\left|\int_{-2}^{0} f(x)dx\right| + \int_0^1 f(x)dx$

7 The solutions to $\sqrt{2} sin\, x = -1$ for $0 \leq x \leq 2\pi$ are:

(A) $\frac{3\pi}{4}$ and $\frac{5\pi}{4}$

(B) $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$

(C) $\frac{5\pi}{4}$ and $\frac{7\pi}{4}$

(D) $\frac{7\pi}{4}$ and $\frac{9\pi}{4}$

8 The solution to the inequality $6 - x - x^2 \leq 0$ is:

(A) $-3 \leq x \leq 2$

(B) $x \leq -3$ or $x \geq 2$

(C) $x \leq -2$ or $x \geq 3$

(D) $-2 \leq x \leq 3$

9 The graph of $y = 3x^2 - kx + 2$ is symmetrical about the line $x = \frac{1}{2}$.
The lowest possible value of y is:

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) $\frac{5}{4}$

10 What is the perpendicular distance between the lines $y = 4x + 3$ and $y = 4x + 5$?

(A) $\frac{2}{\sqrt{17}}$

(B) $\frac{3}{\sqrt{17}}$

(C) $\frac{2}{5}$

(D) $\frac{3}{5}$

End of Section I

Section II

90 marks
Attempt questions 11 – 16
Allow about 2 hours 45 minutes for this section

Answer each section on a new page

In questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new page.

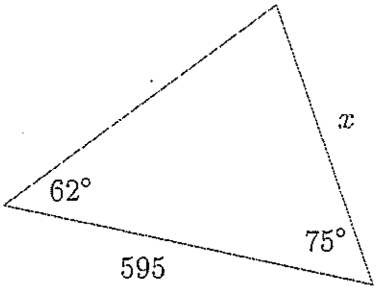
a) Simplify fully $3x - (4 - 3x)$. **1**

b) Factorise $x^3 + 8$ **1**

c) Solve $|2 - 5x| \leq 7$ **2**

d) Write $\frac{3-\sqrt{5}}{3+\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are rational. **2**

e) The side lengths of the triangle below are in millimetres.



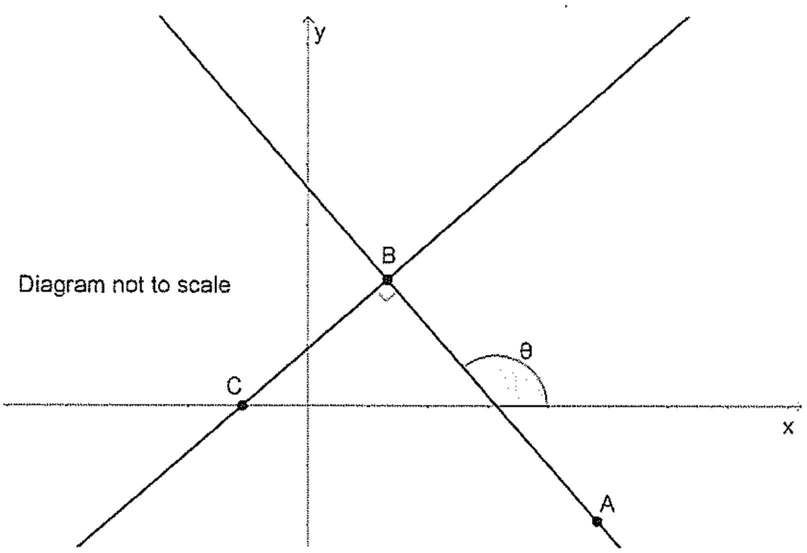
Not to scale

Find the value of x to the nearest whole number. **2**

Question 11 continues on the next page

Question 11 continued

f) The points $A(8,-3)$ and $B(5,4)$ are shown in the diagram below. The line through AB makes an angle of θ with the positive $x-$ axis and the point C lies on the $x-$ axis.



(i) Find the gradient of the line AB. **1**

(ii) Find the value of θ to the nearest degree. **1**

(iii) Find the coordinates of C given that $AB \perp BC$. **2**

(iv) Find coordinates of M, the midpoint of AB. **1**

(v) Find the equation of the line AB in general form. **2**

End of question 11

Question 12 (15 marks) Start a new page.

a) Find the gradient of the normal at the point $(2, -2)$ on the curve $y = x^3 - 5x$. **2**

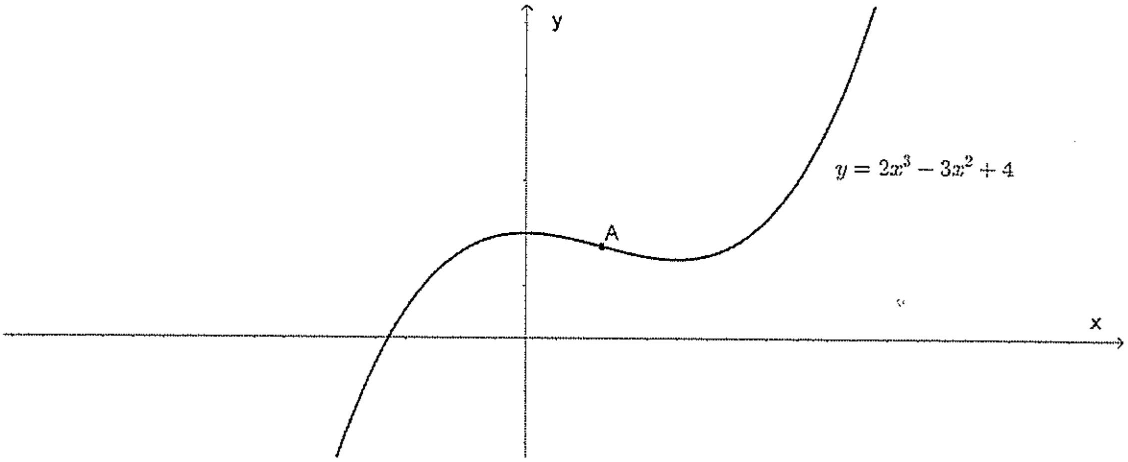
b) Differentiate with respect to x.

(i) $xlog_e(3x^2 - 1)$ **2**

(ii) $(e^{-2x} + 1)^{10}$ **2**

(iii) $\frac{5x}{sin2x}$ **2**

c) The graph below shows the curve $y = 2x^3 - 3x^2 + 4$. The point A is a point of inflexion.



(i) Find the coordinates of A. **2**

(ii) When is the curve concave up? **1**

d) The number of bacteria (B) in a sample grows exponentially with time according to the equation $B = 200e^{kt}$, where k is a constant and t is measured in hours.

(i) In two days (*48* hours) the number of bacteria in the sample is now *1653*. Calculate the value of k to three decimal places. **2**

(ii) Find, correct to the nearest hour, when there will be one million bacteria in the sample. **2**

End of question 12

Question 13 (15 marks) Start a new page.

a) The point $(-2, 3)$ lies on the curve with a gradient function of $\frac{dy}{dx} = \frac{4}{x+3}$.
Find the equation of the curve. **2**

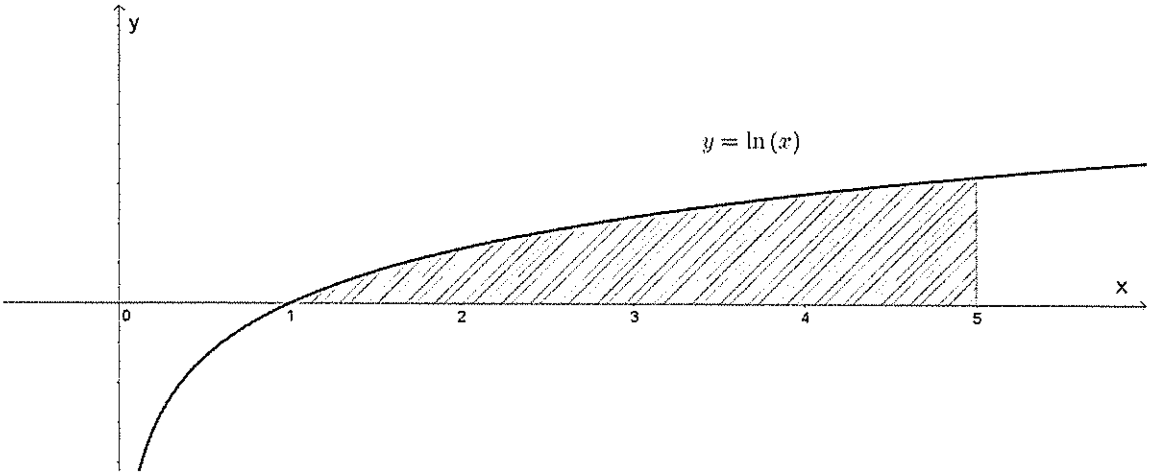
b) Find the following integrals.

(i) $\int (6cos3x - 2sin\frac{x}{2})\,dx$ **2**

(ii) $\int \frac{6}{e^{3x}}\,dx$ **2**

(iii) $\int (1 - 6sec^2\frac{x}{3})dx$ **2**

c) The graph below shows the curve $y = log_e x$.

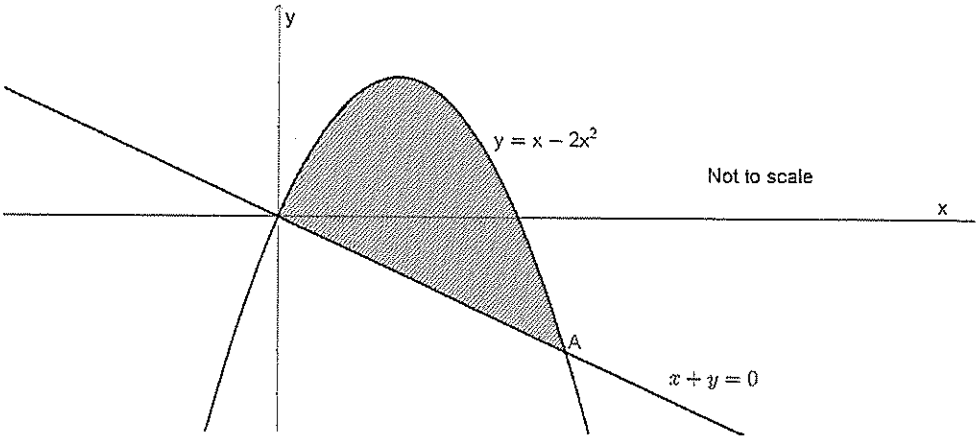


Use Simpson's Rule with five (5) function values to approximate $\int_1^5 log_e x\,dx$. Give your answer to three (3) significant figures. **3**

Question 13 continues on the next page

Question 13 continued

d) The graph below shows the area enclosed by the parabola $y = x - 2x^2$ and the line $x + y = 0$. The parabola and the line intersect at the origin and point A.



(i) Find the coordinates of point A. **1**

(ii) Find the value of the shaded area. **3**

End of question 13

Question 14 (15 marks) Start a new page.

a) An arithmetic sequence begins with the three terms $-6, 1, 8.$

(i) Find the 200th term of the sequence. **2**

(ii) Find the sum of 200 terms of the sequence. **2**

b) A geometric sequence begins with the three terms $-4, 8, -16.$
Find the *15th* term of the progression. **2**

c) The numbers p, q and r add to *9* and form an arithmetic progression. The numbers r, p and q form a geometric progression. Find the values of p, q and r. **3**

d) On the *1st* January each year Simone invests *$M* annually into a superannuation account. The account gives interest at a rate of *5%* per annum, compounded annually.

(i) Show that the value of her investment at the end of *2* years was $A_2 = 2.1525M$ dollars. **2**

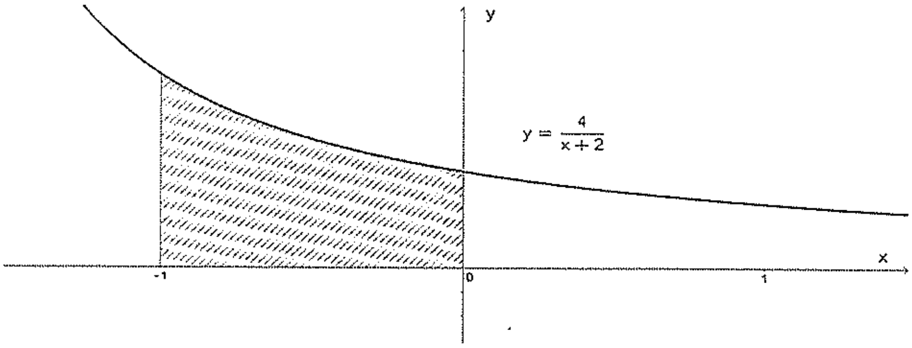
(ii) Show that the value of her investment at the end of n years was $A_n = 21(1.05^n - 1)M$ dollars. **2**

(iii) Simone wants to retire after 30 years with a million dollars in her superannuation account. Find the amount that she must invest into her account on the *1st* January each year to reach her goal. Answer to the nearest cent. **2**

End of question 14

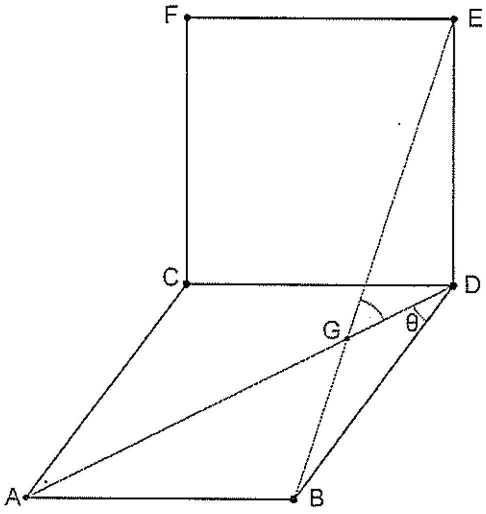
Question 15 (15 marks) Start a new page.

a) The region bounded by the curve $y = \frac{4}{x+2}$, the line $x = -1$ and the axes is shown below.



The region is rotated about the x – axis to form a solid.
Find the volume of this solid. **3**

b) The diagram below shows square $CDEF$ and rhombus $ABDC$.
The diagonal of the rhombus AD and the segment BE intersect at point G.



(i) Given that $< ADB = \theta$, explain why $< CDA = \theta$, giving reasons. **1**

(ii) Find $< BED$ in terms of θ, giving reasons. **2**

(iii) Hence show that $< DGE = \frac{\pi}{4}$, giving reasons. **2**

Question 15 continues on the next page

c) In a large country town, it is known that *55%* of the population is male and *45%* of the population is female. Three people in the town are surveyed at random. Find to the nearest percent, the probability that two are male and one is female. **2**

d) Albert plays a game where he throws two standard six-sided dice and the total of the faces showing is noted. Albert wins the game if an *8* is thrown and he loses if a *5* is thrown. If the sum is any other number, the game continues until an *8* is thrown or a *5* is obtained.

(i) Show that the probability that Albert wins on the first throw is $\frac{5}{36}$. **1**

(ii) Show that the probability that Albert wins on either the first, second or third throw is $\frac{185}{576}$. **2**

(iii) What is the probability that Albert wins the game? **2**

End of question 15

Question 16 (15 marks) Start a new page.

a) Two particles P and Q which are initially at the origin are moving along a straight line. Their displacements, x kilometres, from the origin at any time, t hours, are given by the rules:

$$P:\ x = 5t - 2t^2.$$
$$Q:\ x = 8t^2 + 2t.$$

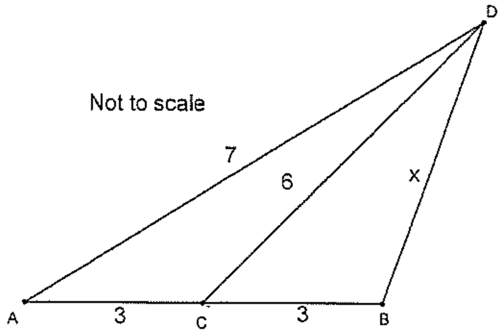
(i) After what time are they travelling with the same velocity? **2**

(ii) Both particles are together again at point A. Find the distance of point A from the origin. **2**

(iii) A third particle R, travelling with constant speed, is 3 kilometres ahead of P and Q when they pass the origin. If particle R arrives at point A at the same time as particles P and Q, find a rule connecting x and t for this particle. **2**

b) Triangle ABD has side lengths of $AD = 7$ units, $DB = x$ units and $AB = 6$ units.

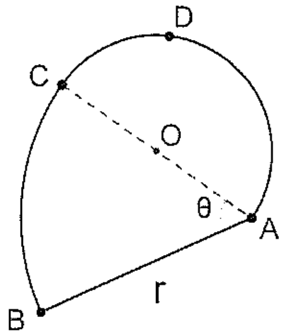
C is the midpoint of AB. The median CD equals the length of the base AB.



(i) Use the cosine rule in triangle ADC to show that $\cos < DAB = \frac{11}{21}$. **1**

(ii) Hence find the exact value of x. **2**

c) The shape $ABCD$ consists of a sector ABC of radius r and angle θ and semicircle ACD with centre O and radius $\frac{r}{2}$.



(i) If the area (A) of this shape is a fixed value, show that the perimeter $P = \left(\frac{\pi+4}{4}\right)r + \frac{2A}{r}$. 3

(ii) Show that perimeter P is a minimum when $\theta = 1$. 3

End of examination

2016

TRIAL MATHEMATICS

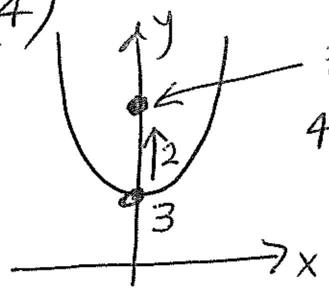
(1) A

(2) D

(3) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-b/a}{c/a}$

$=\frac{5/2}{-1/2}=-5$ C

(4)



Focus (0,5)

$4a=8$

$a=2$ A

(5) $x=a\left(b-\frac{1}{y}\right)$

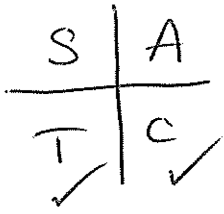
$\frac{x}{a}=b-\frac{1}{y}$

$\frac{1}{y}=b-\frac{x}{a}=\frac{ab-x}{a}$

$y=\frac{a}{ab-x}$ B

(6) B

(7) $\sin x=-1/\sqrt{2}$

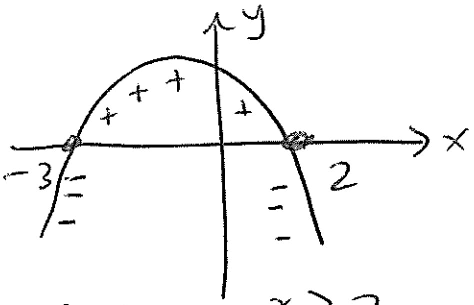


$x=\pi+\pi/4\,,\ 2\pi-\pi/4$

$x=\frac{5\pi}{4}\,,\ \frac{7\pi}{4}$ C

(8) $6-x-x^2\leq 0$

$(3+x)(2-x)\leq 0$



$x\leq -3$ $\quad x\geq 2$ B

(9) $\frac{dy}{dx}=0$ at vertex when $x=1/2$

$6x-k=0$

$6\times\frac{1}{2}-k=0$

$k=3$

$\therefore y=3x^2-3x+2$

$\therefore y=3\left(\frac{1}{2}\right)^2-3\left(\frac{1}{2}\right)+2=1\frac{1}{4}$ D

(10) (0,3) ON $y=4x+3$

DISTANCE TO $4x-y+5=0$

IS $\left|\frac{4\times0-3+5}{\sqrt{4^2+(-1)^2}}\right|=\frac{2}{\sqrt{17}}$

A

QUESTION 11

(a) $3x-4+3x=6x-4$

(b) $(x+2)(x^2-2x+4)$

(c) $2-5x\leq 7$, $-(2-5x)\leq 7$

$-5x\leq 5$ $\quad -2+5x\leq 7$

$x\geqslant -1$ $\quad x\leq 1\cdot 8$

i.e. $-1<x<1\cdot 8$

(d) $\dfrac{3-\sqrt{5}}{3+\sqrt{5}}\times\dfrac{3-\sqrt{5}}{3-\sqrt{5}}$

$=\dfrac{9-6\sqrt{5}+5}{9-5}$

$=\dfrac{14-6\sqrt{5}}{4}$

$=\dfrac{7}{2}-\dfrac{3}{2}\sqrt{5}$

$a=7/2$ $\quad b=-3/2$

(e) $\dfrac{x}{\sin 62^\circ}=\dfrac{595}{\sin 43^\circ}$

$x\doteqdot 770$

(f)(i) $m_{AB}=\dfrac{-3-4}{8-5}$

$=-\dfrac{7}{3}$

(ii) $\tan\theta=-7/3$

$\theta=180^\circ-67^\circ=113^\circ$

(iii) Let C be $(c,0)$

$m_{BC}\cdot m_{AB}=-1$

$\dfrac{0-4}{c-5}\cdot\dfrac{-7}{3}=-1$

$28=-3(c-5)$

$28=-3c+15$

$13=-3c$

$c=-4\,1/3$

$\therefore C\left(-4\frac{1}{3},0\right)$

(iv) $\left(\dfrac{8+5}{2},\dfrac{-3+4}{2}\right)=\left(6\frac{1}{2},\frac{1}{2}\right)$

(v) $y-y_1=m(x-x_1)$

$y-4=-\frac{7}{3}(x-5)$

$3(y-4)=-7(x-5)$

$3y-12=-7x+35$

$7x+3y-47=0$

QUESTION 12

(a) $y = x^3 - 5x$

$\frac{dy}{dx} = 3x^2 - 5$

$\underline{x=2}$ $\quad \frac{dy}{dx} = 3(2)^2 - 5 = 7$

$m_1 = 7$

∴ Gradient of normal

$m_2 = -\frac{1}{m_1} = -\frac{1}{7}$

(b) Product Rule

(i) $x \frac{d}{dx}\left(\ln(3x^2-1)\right) + \ln(3x^2-1)\frac{d}{dx}(x)$

$= x\left(\frac{6x}{3x^2-1}\right) + \ln(3x^2-1)\ 1$

$= \frac{6x^2}{3x^2-1} + \ln(3x^2-1)$

(ii) Chain Rule

$10(e^{-2x}+1)^9 \frac{d}{dx}(e^{-2x}+1)$

$= 10(e^{-2x}+1)^9(-2e^{-2x})$

$= -20e^{-2x}(e^{-2x}+1)^9$

(iii) Quotient Rule

$= \frac{\sin 2x \frac{d}{dx}(5x) - 5x\frac{d}{dx}(\sin 2x)}{(\sin 2x)^2}$

$= \frac{5\sin 2x - 10x\cos 2x}{\sin^2 2x}$

(c) $y = 2x^3 - 3x^2 + 4$

(i) $y' = 6x^2 - 6x$

$y'' = 12x - 6$

Point of Inflexion $y'' = 0$

$12x - 6 = 0$

$x = 1/2$

$y = 2(\frac{1}{2})^3 - 3(\frac{1}{2})^2 + 4 = 3\frac{1}{2}$

∴ $A(\frac{1}{2}, 3\frac{1}{2})$

(ii) Concave up: $y'' > 0$

$x > \frac{1}{2}$

(d) (i) $\underline{t = 48 \quad B = 1653}$

$1653 = 200e^{48k}$

$e^{48k} = 8.265$

$48k = \ln(8.265)$

$k = \frac{\ln(8.265)}{48} = 0.044$

(ii) $\boxed{B = 200e^{0.044t}}$

$1000000 = 200e^{0.044t}$

$e^{0.044t} = 5000$

$t = \frac{\ln 5000}{0.044} \doteqdot 194$ hours

(about 8 days)

QUESTION 13

(a) $y = 4\log_e(x+3) + C$

SUB $(-2, 3)$

$3 = 4 \times 0 + C$

$\therefore C = 3$

EQUATION

$y = 4\log_e(x+3) + 3$

(b)

(i) $-2\sin 3x - 4\cos\frac{x}{2} + C$

(ii) $-2e^{-3x} + C$ or $\frac{-2}{e^{3x}} + C$

(iii) $x - 18\tan\frac{x}{3} + C$

(c) $\int_1^5 \log_e x\,dx = \int_1^3 \log_e x\,dx + \int_3^5 \log_e x\,dx$

$= \frac{3-1}{6}(\log_e 1 + 4\log_e 2 + \log_e 3)$

$+ \frac{5-3}{6}(\log_e 3 + 4\log_e 4 + \log_e 5)$

$= 1.2904 + 2.7510$

$\doteqdot 4.04$

Solve simultaneously

(d) $\begin{cases} y = -x \\ y = x - 2x^2 \end{cases}$

(i) $-x = x - 2x^2$

$0 = 2x - 2x^2$

$0 = 2x(1-x)$

$\left.\begin{array}{l} x = 0 \\ y = 0 \end{array}\right\}$ $\left.\begin{array}{l} x = 1 \\ y = -1 \end{array}\right\}$

So $A = (1, -1)$

(ii) Area $= \int \left(\text{top graph} - \begin{array}{c}\text{bottom} \\ \text{graph}\end{array}\right)$

$= \int_0^1 (x - 2x^2) - (-x)\,dx$

$= \int_0^1 2x - 2x^2\,dx$

$= \left[x^2 - \frac{2}{3}x^3\right]_0^1$

$= (1 - 2/3) - 0$

$= \frac{1}{3}$ units^2

QUESTION 14

(a) $a = -6 \quad d = 7$

(i) $T_n = a + (n-1)d$

$T_{200} = -6 + 199 \times 7$

$T_{200} = 1387$

(ii) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{200} = \frac{200}{2}[-12 + 199 \times 7]$

$S_{200} = 138100$

(b) $T_n = ar^{n-1}$

$T_{15} = -4(-2)^{14}$

$T_{15} = -65536$

(c) p, q, r form an AP

$\therefore q - p = r - q$

$2q = p + r$

Also $q + p + r = 9$

$q + 2q = 9$

$3q = 9$

$q = 3$ --- ①

and $p + r = 6$ -- ②

r, p, q form a GP

$$\frac{p}{r} = \frac{q}{p}$$

$p^2 = rq$

Using ① & ②

$p^2 = (6-p)3$

$p^2 = 18 - 3p$

$p^2 + 3p - 18 = 0$

$(p+6)(p-3) = 0$

$$\begin{cases} p = -6 \\ q = 3 \\ r = 12 \end{cases} \qquad \begin{cases} p = 3 \\ q = 3 \\ r = 3 \end{cases}$$

(d) $A_0 = M$

(i) $A_1 = M \times 1.05$

$$A_2 = (M \times 1.05 + \underset{\substack{\uparrow \\ \text{NEXT} \\ \text{INVESTMENT}}}{M}) \times \underset{\substack{\uparrow \\ \text{INTEREST}}}{1.05}$$

$A_2 = M \times 1.05^2 + M \times 1.05$

$A_2 = (1.05^2 + 1.05)M$

$A_2 = 2.1525M$

(ii) $$A_3 = (A_2 + \underset{\substack{\uparrow \\ \text{NEXT} \\ \text{INVESTMENT}}}{M}) \times \underset{\substack{\uparrow \\ \text{INTEREST}}}{1.05}$$

$= (M \times 1.05^2 + M \times 1.05 + M) \times 1.05$

$= M \times 1.05^3 + M \times 1.05^2 + M \times 1.05$

$= (1.05^3 + 1.05^2 + 1.05)M$

Continuing the pattern

$A_n = (1.05^n + 1.05^{n-1} + \cdots + 1.05)M$

↑

GP n terms

$a = 1.05$

$r = 1.05$

$$A_n = \frac{a(r^n - 1)}{r - 1} M$$

$$= \frac{1.05(1.05^n - 1)}{1.05 - 1} M$$

$$= \frac{1.05}{0.05}(1.05^n - 1)M$$

$$= 21(1.05^n - 1)M$$

(III) $A_n = 1000000 \quad n = 30$

$$1000000 = 21(1.05^{30} - 1)M$$

$$M = \frac{1000000}{21(1.05^{30} - 1)}$$

$$M = 14334.70$$

So each investment needs to be $14334.70

QUESTION 15

15(a) $V = \pi \int y^2 \, dx$

$$= \pi \int_{-1}^{0} \frac{16}{(x+2)^2} \, dx$$

$$= 16\pi \int_{-1}^{0} (x+2)^{-2} \, dx$$

$$= 16\pi \left[-(x+2)^{-1} \right]_{-1}^{0}$$

$$= 16\pi \left[\frac{-1}{x+2} \right]_{-1}^{0}$$

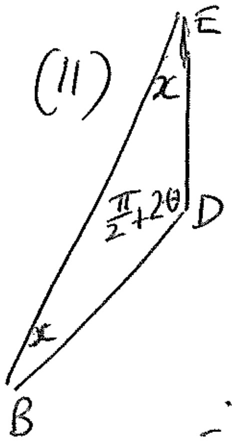
$$= 16\pi \left[(-\tfrac{1}{2}) - (-1) \right]$$

$$= 8\pi \text{ units}^3$$

(b)(i) $\angle CDA = \angle ADB = \theta$

(The digonal of rhombus bisects the vertex angle)

(ii)



$\angle EDC = \frac{\pi}{2}$ ($\angle$ in a square)

$\therefore \angle EDB = \frac{\pi}{2} + 2\theta$

$DE = CD$ (sides of square)

$BD = CD$ (sides of rhombus)

$\therefore DE = BD$

$\therefore \angle BED = \angle EDB$ ($\angle$'s opposite equal sides in ΔBDE)

$x + x + (\frac{\pi}{2} + 2\theta) = \pi$

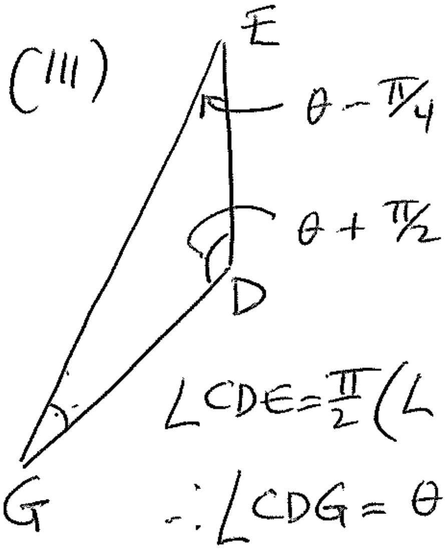
(angle sum of Δ)

$2x + 2\theta = \frac{\pi}{2}$

$x + \theta = \frac{\pi}{4}$

$\therefore \angle BED = x = \frac{\pi}{4} - \theta$

(iii)



$\angle CDE = \frac{\pi}{2}$ (∠ in square)

$\therefore \angle CDG = \theta + \pi/2$

$\angle DGE + \left(\frac{\pi}{4} - \theta\right) + \left(\theta + \frac{\pi}{2}\right) = \pi$

(∠ sum of △DGE)

$\angle DGE + \frac{3\pi}{4} = \pi$

$\angle DGE = \pi/4$

(b) $P(MMF) + P(MFM) + P(FMM)$

$= 3 \times \cdot 55 \times \cdot 55 \times \cdot 45$

$\doteqdot 41\%$

(c)

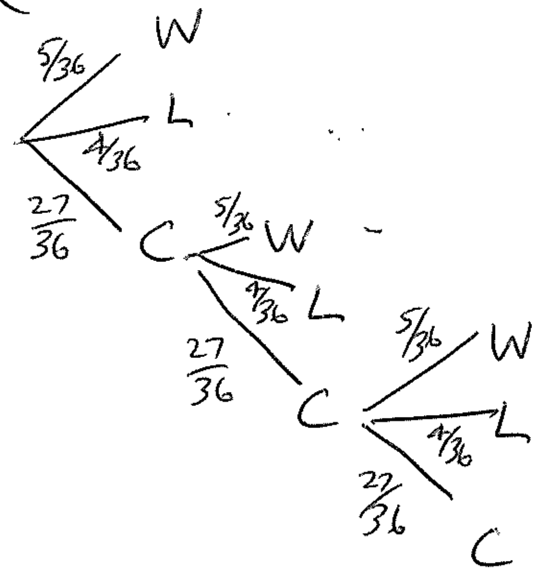
(i)

	1	2	3	4	5	6
1	2	3	4	⑤	6	7
2	3	4	⑤	6	7	⑧
3	4	⑤	6	7	⑧	9
4	⑤	6	7	⑧	9	10
5	6	7	⑧	9	10	11
6	7	⑧	9	10	11	12

$P(8) = 5/36$

∴ W = WIN L = LOSE C = CONTINUE

(ii)



$P = \frac{5}{36} + \frac{3}{4} \times \frac{5}{36} + \frac{3}{4} \times \frac{3}{4} \times \frac{5}{36}$

$= \frac{5}{36}\left(1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2\right)$

$= 185/576$

(iii) Continuing pattern above indefinitely

$P = \frac{5}{36}\left(1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \cdots\right)$

LIMITING SUM

$= \frac{5}{36}\left(\frac{a}{1-r}\right)$

$= \frac{5}{36}\left(\frac{1}{1-3/4}\right)$

$= \frac{5}{9}$

QUESTION 16

(a) $P: V = 5 - 4t$

(i) $Q: V = 16t + 2$

$$5 - 4t = 16t + 2$$

$t = \frac{3}{20}$ hour (9 mins)

(ii) $5t - 2t^2 = 8t^2 + 2t$

$$10t^2 - 3t = 0$$

$$t(10t - 3) = 0$$

$t = 0$ (at origin) $\quad t = \frac{3}{10}$ hour (at A)

$\therefore x = 5\left(\frac{3}{10}\right) - 2\left(\frac{3}{10}\right)^2 = 1.32$

So A is 1.32 km from origin

(iii) Particle R $\quad V = C_1$

$$X = C_1 t + C_2$$

$t = 0 \quad x = 3 \quad \therefore C_2 = 3$

$t = \frac{3}{10} \quad X = 1.32 \quad 1.32 = C_1\left(\frac{3}{10}\right) + 3$

$$C_1 = -5.6$$

$$\therefore x = -5.6t + 3$$

(b)(i) $\cos\angle DAB = \frac{3^2 + 7^2 - 6^2}{2 \times 3 \times 7} = \frac{11}{21}$

(ii) $x^2 = 7^2 + 6^2 - 2 \times 6 \times 7 \times \cos\angle DAB$

$$x^2 = 7^2 + 6^2 - 2 \times 6 \times 7 \times 11/21$$

$$x = \sqrt{41}$$

(c)(i) $A =$ sector + semicircle

$$A = \frac{1}{2}r^2\theta + \frac{1}{2}\pi\left(\frac{1}{2}r\right)^2$$

$$A = \frac{1}{2}r^2\theta + \frac{1}{8}\pi r^2$$

$$8A = 4r^2\theta + \pi r^2$$

$$4r^2\theta = 8A - \pi r^2 \quad (\div 4r^2)$$

$$\theta = \frac{2A}{r^2} - \frac{\pi}{4} \quad \ldots\ldots *$$

$P =$ radius + arc + semicircle

$$P = r + r\theta + \frac{1}{2} \times 2\pi\left(\frac{1}{2}r\right)$$

$$P = r + r\left(\frac{2A}{r^2} - \frac{\pi}{4}\right) + \frac{\pi r}{2}$$

$$P = r + \frac{2A}{r} - \frac{\pi r}{4} + \frac{\pi r}{2}$$

$$P = \frac{2A}{r} + r\left(1 + \frac{\pi}{4}\right)$$

$$P = \frac{2A}{r} + \left(\frac{4+\pi}{4}\right)r$$

(ii) $\frac{dP}{dr} = \frac{-2A}{r^2} + \left(\frac{4+\pi}{4}\right)$

$$\frac{d^2P}{dr^2} = \frac{4A}{r^3} > 0$$

$\therefore \frac{dP}{dr} = 0$ GIVES MINIMUM

$$\frac{2A}{r^2} = \frac{4+\pi}{4}$$

$r = \sqrt{\frac{8A}{4+\pi}}$ since $r > 0$

$\therefore \theta = \frac{2A}{\frac{8A}{4+\pi}} - \frac{\pi}{4}$ from *

$\theta = \frac{4+\pi}{4} - \frac{\pi}{4} = 1$

(radian)